AP® Calculus AB
2001 Sample Student Responses

The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

A1

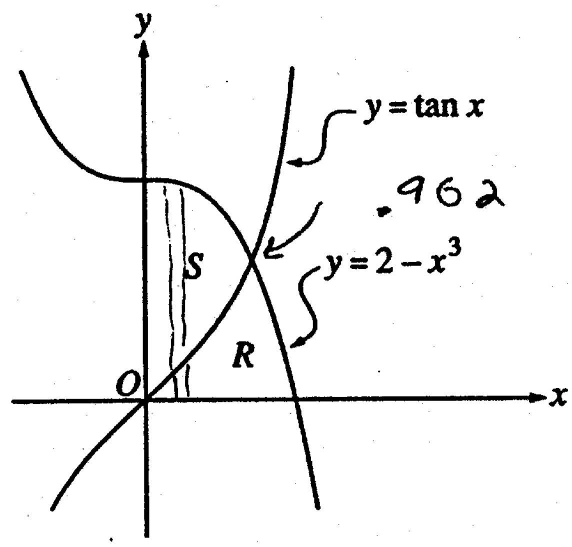
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



$(.902, 1.266)$

$\sqrt[3]{2-y}$

Work for problem 1(a)

Area of R: $\int_{bottom}^{top} [Right(y) - Left(y)]\,dy$

$\int_{0}^{1.266} \left[\sqrt[3]{2-y} - \tan^{-1} y\right] dy$

Area of R = .729

$\tan x + x^3 = 2$

$x = .902$

Work for problem 1(b)

Area of S: $\int_{Left}^{Right} [top(x) - bottom(x)]\, dx$

$\int_0^{.902} [(2-x^3) - \tan x]\, dx$

Area of S = 1.161

Work for problem 1(c)

Volume of S about the x-axis:

$\pi \int R(x)^2\, dx$

$\pi \int_0^{.902} [(2-x^3)^2 - \tan^2 x]\, dx$

Volume of S about the x-axis = 8.332

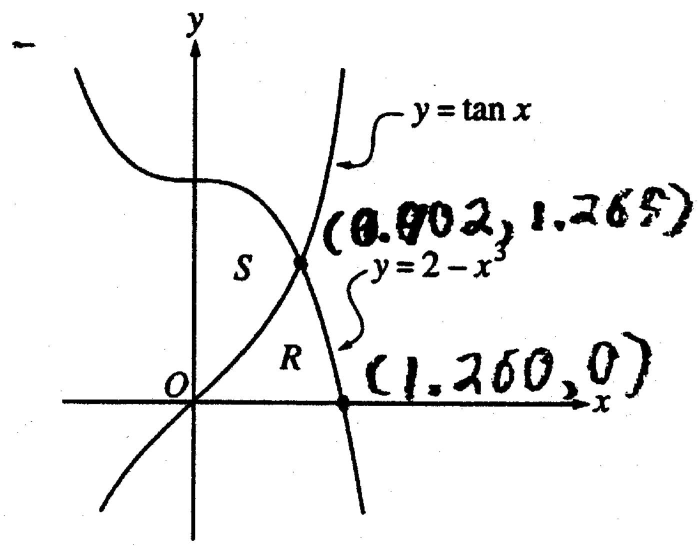
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$A_2 = \int_0^{0.902} \tan x \, dx + \int_{0.902}^{1.260} 2 - x^3 \, dx =$$

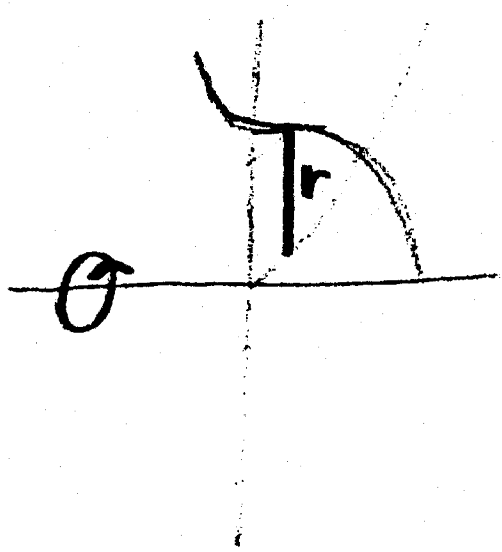
$$0.478 + 0.251 = 0.729$$

Work for problem 1(b)

$$A_s = \int_0^{0.902} (2-x^3) - \tan x \, dx$$

$$A_s = 1.160$$

Work for problem 1(c)



$$V = \int_0^{0.902} \pi ((2-x^3) - \tan x)^2 \, dx$$

$$V = 5.555$$

AP® Calculus AB
2001 Sample Student Responses

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

A1

Work for problem 2(a)

$$\left.\frac{dW}{dt}\right|_{t=12} \approx \left.\frac{\Delta W}{\Delta t}\right|_{9<t<15} = \frac{24-21}{9-15} = -.5\,°C/\text{day}$$

Work for problem 2(b)

$$\text{Average } W = \frac{\int^{15} W\,dt}{\Delta t} = \frac{\int_0^{15} W\,dt}{15} \approx \frac{\left(\frac{20+31}{2} + \frac{31+28}{2} + \frac{28+24}{2} + \frac{24+22}{2} + \frac{22+21}{2}\right)3}{15}$$

bases height

$$A_{trap} = \frac{b_1+b_2}{2}\cdot h$$

$h=3$

$$A_{ve_W} = 25.1\,°C$$

A₂

Work for problem 2(c)

$$P'(t) = 10e^{-\frac{t}{3}} + 10t \cdot \tfrac{-1}{3} e^{-\frac{t}{3}}$$

$$P'(12) = 10e^{-\frac{12}{3}}\left(1 + \tfrac{-1}{3}t\right)$$

$$-30e^{-4} = -.5495°C/\text{day}$$

On the 12th day in the 15 day time period, the temperature of the pond will be decreasing at such a rate, that it will be getting about .5495 degrees C colder per day.

Work for problem 2(d)

$$\text{Average } P = \frac{\int_0^{15} P\,dt}{\Delta t} = \frac{\int_0^{15} 20 + 10te^{-\frac{1}{3}t}\,dt}{15}$$

$$= \frac{\left.20t\right|_0^{15} + 10\int_0^{15} te^{-\frac{1}{3}t}\,dt}{15}$$

$$\int te^{-\frac{1}{3}t}\,dt \qquad u = t \quad v = -3e^{-\frac{1}{3}t} \qquad du = dt \quad dv = e^{-\frac{1}{3}t}\,dt$$

$$= -3te^{-\frac{1}{3}t} + 3\int e^{-\frac{1}{3}t}\,dt$$

$$= -3te^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t} + C$$

$$= -3e^{-\frac{1}{3}t}(t+3) + C$$

$$= \frac{300 + 10\left.\left[-3e^{-\frac{1}{3}t}(t+3)\right]\right|_0^{15}}{15}$$

$$= \frac{300 + 86.3616}{15} = 25.757°C$$

C₁

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

Work for problem 2(a)

$$W'(12) \approx \frac{w(9) - w(15)}{9-15}$$

$$\approx \frac{24 - 21}{9-15}$$

$$W'(12) \approx -\frac{1}{2}$$

Work for problem 2(b)

$$\text{Ave } W(t) = \frac{1}{2} \cdot (3)\left(\frac{W(0) + 2W(3) + 2W(6) + 2W(9) + 2W(12) + W(15)}{15-0}\right)$$

$$\text{Ave } W(t) = \frac{1}{10}\left[20 + (2 \cdot 31) + (2 \cdot 28) + (2 \cdot 24) + (2 \cdot 22) + 21\right]$$

$$\text{Ave } W(t) = \frac{1}{10}(251)$$

$$\text{Ave } W(t) = 25.1\ °C$$

C_2

Work for problem 2(c)

$P'(t) = 10e^{(-t/3)} - \frac{10}{3}te^{(-t/3)}$

$P'(12) = 10e^{-12/3} - (10/3)(12)(e^{(\frac{-12}{3})})$

$P'(12) = -.5495$ degrees/day

P'(12) is the instantaneous rate of change of degrees in celcius per day, when t=12 days.

Work for problem 2(d)

$\text{Ave } P(t) = \frac{1}{15-0}\int_0^{15}\left[20 + 10te^{(-t/3)}\right]dt$

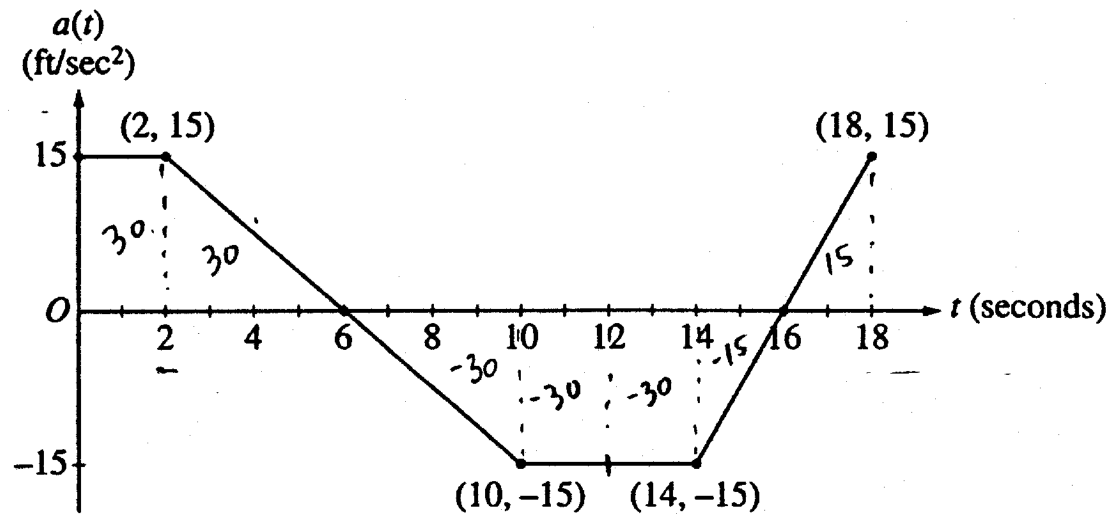
$\text{Ave } P(t) = \frac{1}{15}(386.362)$

$\text{Ave } P(t) = 25.757\ °C$

AP® Calculus AB
2001 Sample Student Responses

The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

A,



Work for problem 3(a)

Yes, the velocity is increasing at t=2s because according to the graph, the car is accelerating at that time. Acceleration is positive ($a_{t=2} = 15$ ft/s²), thus the car must be getting faster.

Work for problem 3(b)

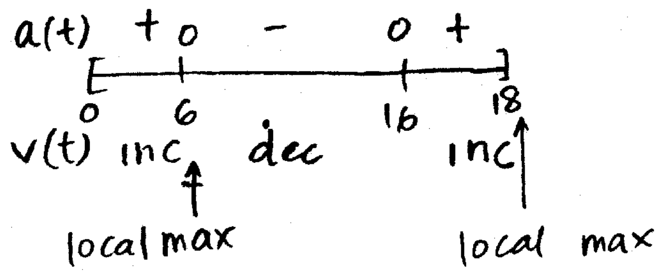
$v = \int_0^t a\,dt$

∴ velocity at time x = area under graph between t=0 and t=x +55 (initial velocity)

∴ v = 55 when area under graph is zero

v = 55 ft/s at t = 12

Work for problem 3(c)



$V_6 = \int_0^6 a(t)\,dt + 55$

$= 60 + 55$

$= 115$ ft/s

$V_{18} = \int_0^{18} a(t)\,dt + 55$

$= -30 + 55$

$= 25$ ft/s

∴ absolute max velocity = 115 ft/s
at it occurs at $t = 6$s

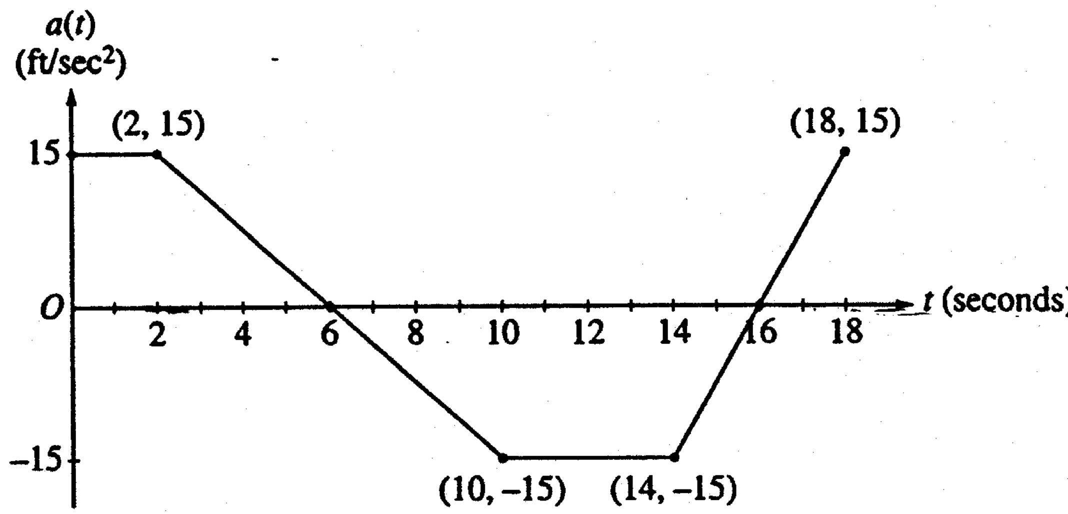
Work for problem 3(d)

$V = \int_0^x a(t)\,dt + 55$

$-55 = \int_0^x a(t)\,dt$

for velocity to be zero, the area under the graph must be −55, which does not occur between $t = 0$s and $t = 18$s, as shown by the graph.
The lowest velocity on the interval $0 \le t \le 18$ is at $t = 16$, and it is 10 ft/s.

D1



Work for problem 3(a)

At $t=2$ seconds, the acceleration of the car is positive $15 \frac{ft}{sec^2}$ $\therefore$

yes, the velocity of the car is increasing

Work for problem 3(b)

$v(t) = \int_0^x a(t)\,dt + 55 \frac{ft}{sec}$

$55 \frac{ft}{sec} = \int_0^x a(t)\,dt + 55 \frac{ft}{sec}$

$0 = \int_0^x a(t)\,dt$

from graph $\int_0^{12} a(t)\,dt = 0 \quad \therefore \quad v(12) = 55 \frac{ft}{sec}$

$2(15) + \frac{1}{2}(4)(15)$

$-6(15) + \frac{1}{2}(4)(15) = 0$

$\boxed{t=12}$

D2

Work for problem 3(c)

$v'(t) = a(t)$

from graph

$v'(t)$ pos. $[0, 6)$

$v'(t) = 0$ $t = 6$

$v'(t)$ neg. $(6, 16)$ --

$v'(t) = 0$ $t = 16$

$v'(t)$ pos $(16, 18]$

A maximum occurs at $t = 6$ because the acceleration switches from positive to negative at that point and continues to be negative for an interval longer than it is positive.

$$v(t) = \int_0^x a(t)\,dt + 55\frac{ft}{sec}$$

$$v(6) = \int_0^6 a(t)\,dt + 55\frac{ft}{sec}$$

$$= 55\frac{ft}{sec} + 2sec\left(15\frac{ft}{sec^2}\right) + \frac{1}{2}(4\ sec)\left(15\frac{ft}{sec^2}\right) = \boxed{115\frac{ft}{sec}}$$

Work for problem 3(d)

$$v(t) = \int_0^x a(t)\,dt + 55\frac{ft}{sec}$$

$$0 = \int_0^x a(t)\,dt + 55\frac{ft}{sec}$$

$-55\frac{ft}{sec} \neq \int_0^x a(t)\,dt$ from graph, the $\int a(t)\,dt$ never equals $55\frac{ft}{sec}$

the velocity is never equal to zero

AP® Calculus AB
2001 Sample Student Responses

The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

NO CALCULATOR ALLOWED

A1

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$h'(x) = 0 = \frac{x^2-2}{x}$

$0 = x^2 - 2$

$x = \pm\sqrt{2}$

$\therefore$ h has horizontal tangents at $x = \sqrt{2}$ and $x = -\sqrt{2}$

$h'(x)$: $-$ 0 $+$ U $-$ 0 $+$ on a number line marked $-\sqrt{2}$, 0, $\sqrt{2}$

$\therefore$ h has a local minimum at $x = \sqrt{2}$ and $x = -\sqrt{2}$

Work for problem 4(b)

$$h''(x) = \frac{2x \cdot x - (x^2-2)}{x^2}$$

$$= \frac{2x^2 - x^2 + 2}{x^2}$$

$$= \frac{x^2+2}{x^2}$$

$h''(x)$: $+$ U $+$ on a number line marked 0

$\therefore$ h is concave up on the intervals $(-\infty, 0)$ and $(0, \infty)$

A2

Work for problem 4(c)

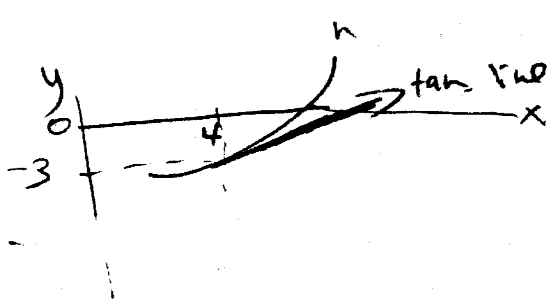
$h'(4) = \frac{4^2-2}{4} = \frac{14}{4} = \frac{7}{2}$

$h(4) = -3$

$y + 3 = \frac{7}{2}(x - 4)$

Work for problem 4(d)

The tangent line to the graph of h at x = 4 lies below the graph of h for x > 4 because h is concave up on the interval (0, ∞).



NO CALCULATOR ALLOWED

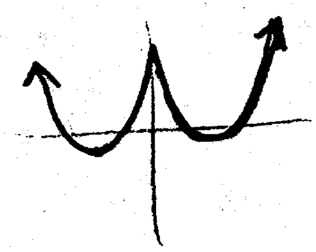
D1

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3



No calculator is allowed for these problems.

Work for problem 4(a)

$1 - \frac{2}{x^2} : x$

$x - \frac{2}{x}$

$h'(x) = 0$

$x \neq 0 \qquad \frac{x^2 - 2}{x} = 0$

$x - \frac{2}{x}$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$h''(x) = \frac{2x^2 - x^2 + 2}{x^2}$

$h''(+\sqrt{2}) = \frac{4 - 2 + 2}{2}$

$h''(\sqrt{2}) = 4/2 = 2$

$h''(\sqrt{2})\ +$

$h''(-\sqrt{2})\ +$

at $+\sqrt{2}$, minimum

+ at $-\sqrt{2}$, minimum

because the second derivative is +, which means the slope is increasing

Work for problem 4(b)

$x \neq 0$

$\frac{2x^2 - x^2 + 2}{x^2} > 0$

~~$2x^2x$~~

$x^2 + 2 > 0$

$x^2 > -2$

x^2

(h) is concave up for all values, as x^2 must always be greater than -2

NO CALCULATOR ALLOWED

D2

Work for problem 4(c)

$h(4) = -3$

$h'(x) = \frac{x^2 - 2}{x}$

$h'(4) = \frac{16-2}{4} = \frac{14}{4} = \frac{7}{2}$

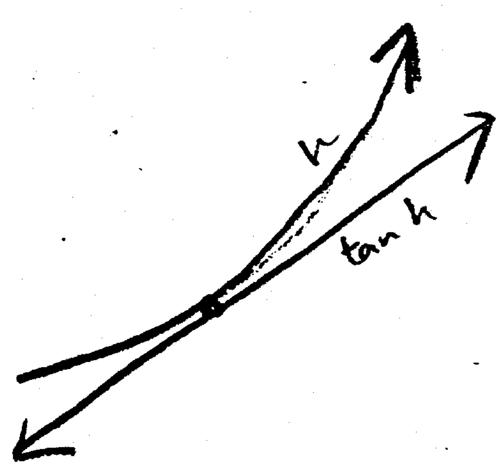
$y + 3 = \frac{7}{2}(x - 4)$

$y = \frac{7}{2}x - 14 - 3$

$y = \frac{7}{2}x - 17$

Work for problem 4(d)

below, because the graph is concaved up



AP® Calculus AB
2001 Sample Student Responses

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

NO CALCULATOR ALLOWED

A1

Work for problem 5(a)

$f(x) = 4x^3 + ax^2 + bx + k$

$f'(x) = 12x^2 + 2ax + b$

$f''(x) = 24x + 2a$

$f''(-2) = 24(-2) + 2a = 0 \rightarrow a = 24$

$f'(-1) = 12(-1)^2 + 2(24)(-1) + b = 0 \rightarrow b = 36$

$12 - 48$

NO CALCULATOR ALLOWED

A_2

Work for problem 5(b)

$\int_0^1 4x^3 + 24x^2 + 36x + k \, dx = 32$

$x^4 + 8x^3 + 18x^2 + kx \Big|_0^1 = 32$

$1 + 8 + 18 + k = 32$

$k = 5$

NO CALCULATOR ALLOWED

Work for problem 5(a)

C_1

$f'(x) = 12x^2 + 2ax + b = 0$

$12(-1)^2 + 2a(-1) + b = 0$

$12 - 2a + b = 0$

$f''(x) = 24x + 2a = 0$

$24(-1) + 2a = 0$

$-24 + 2a = 0$

$a = 12$
$b = -12$

Work for problem 5(b)

C_2

$$\int_0^1 4x^3 + 12x^2 - 12x + K = 32$$

$$\left[\frac{4x^4}{4} + \frac{12x^3}{3} - \frac{12x^2}{2} + Kx\right]_0^1 = 32$$

$$\left[x^4 + 4x^3 - 6x^2 + Kx\right]_0^1 = 32$$

$$1 + 4 - 6 + 1K = 32$$

$$-1 + 1K = 32$$

$$\boxed{K = 33}$$

AP® Calculus AB
2001 Sample Student Responses

The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

A1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$y = f(x)$
$f(3) = \frac{1}{4}$

$\frac{dy}{dx} = y^2(6-2x)$

$\frac{d^2y}{dx^2} = (y^2)(-2) + (6-2x)\cdot(2y\cdot y')$

$\frac{d^2y}{dx^2} = -2y^2 + 12y\cdot y' - 4xy\cdot y'$

$\frac{d^2y}{dx^2}(3,\frac{1}{4}) = -2(\frac{1}{4})^2 + 12(\frac{1}{4})\cdot y' - 4(3)(\frac{1}{4})\cdot y'$

$= -\frac{1}{8} + 3y' - 3y'$

$\frac{d^2y}{dx^2}(3,\frac{1}{4}) = -\frac{1}{8}$

NO CALCULATOR ALLOWED

A_2

Work for problem 6(b)

$$\frac{dy}{dx} = y^2(6-2x)$$

$$\frac{dy}{y^2} = (6-2x)\,dx$$

$$\frac{1}{y^2}\,dy = (6-2x)\,dx$$

$$\int y^{-2}\,dy = \int (6-2x)\,dx$$

$$-y^{-1} = 6x - x^2 + c$$

$$-\frac{1}{y} = 6x - x^2 + c$$

$$\frac{1}{y} = x^2 - 6x - c$$

$$y = \frac{1}{x^2 - 6x + c}$$

$$\frac{1}{4} = \frac{1}{(3)^2 - 6(3) + c}$$

$$\frac{1}{4} = \frac{1}{c-9}$$

$$c - 9 = 4$$

$$c = 13$$

$$y = \frac{1}{x^2 - 6x + 13}$$

NO CALCULATOR ALLOWED

C

Work for problem 6(a)

$(2y^2x)$ $uv' + vu'$

$2y^2 + 4yy'x$

$y' = y^2(6-2x)$

$y' = 6y^2 - 2y^2x$

$y'' = 12yy' - 2y^2 - 4yy'x \leftarrow \frac{d^2y}{dx^2}$

$y'' = 12y(y^2(6-2x)) - 2y^2 - 4y(y^2(6-2x))$ at $(3, \frac{1}{4})$

$y'' = 12(\frac{1}{4})(\frac{1}{8}(6-6)) - 2(\frac{1}{8}) - 4(\frac{1}{4})(\frac{1}{8}(6-6))$

$y'' = 3(0) - \frac{2}{8} - 0$

$y'' = -\frac{2}{8} = -\frac{1}{4}$

$\frac{d^2y}{dx^2}$ at point $(3, \frac{1}{4}) = \boxed{-\frac{1}{4}}$

NO CALCULATOR ALLOWED

C

$f(3) = \frac{1}{4}$

Work for problem 6(b)

$\frac{dy}{dx} = y^2(6-2x)$

$\int \frac{dy}{y^2} = \int 6-2x\,dx$

$\int y^{-2}dy = \int 6-2x\,dx$

$-\frac{y^{-1}}{} = 6x - \frac{2x^2}{2} + C$

$-\frac{1}{y} = 6x - x^2 + C$

$-\frac{1}{\frac{1}{4}} = 6(3) - 9 + C$

$-4 = 18-9+C$

$-4 = -9+C$

$-4+9 = C$

$5 = C$

$-\frac{1}{y} = \frac{6x-x^2+5}{1}$

$-1 = y(6x-x^2+5)$

$\frac{-1}{6x-x^2+5} = y$